

Puzzlesolver's Delight

*a booklet of puzzles, as devised by the students of
Mathematical Insights in Computing*

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compiled by Chelsea Voss (voss.chelsea@gmail.com)

Puzzle 1: Farmer and Dog

Puzzle 2: Two Guards

Puzzle 3: Equilateral Triangle

Puzzle 4: Towers of Hanoi

Puzzle 5: Blue Eyes

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Puzzle 11: Achilles and the Turtle

Puzzle 12: Five Perfectly Logical Pirates

Puzzle 13: Find the Poison

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Puzzle 1.

A farmer has three gold stakes that he has placed in his farm. He wishes to go into town but can't bring the stakes with him, so he wants to find a way to let his dog watch them. What is a way that the farmer can tie his dog with one leash to the three stakes such that the dog cannot run, and if any one of the three stakes is removed the dog will be released to go chase after the thief?

Puzzle 2.

The two guards problem is famous for being especially difficult to people who have never heard it before. Although it is generally well known, this puzzle remains my favorite. There are several variations, but the one I heard goes something like this:

There are two doors, one door leads to Heaven and the other to Hell. In front of each door stands a guard. One guard always tells the truth and one always lies. You do not know which guard is the liar and which one tells the truth. You may ask one question of one guard to

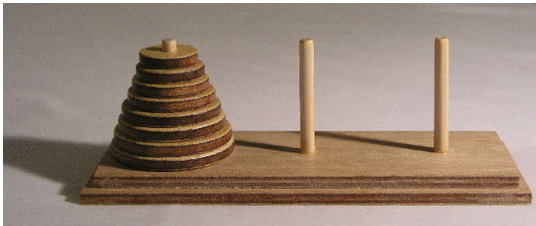
determine which door is the way to Heaven.
What is that question?

Puzzle 3.

A math problem that really got me thinking and had me working for a while to solve it is the following: Say we have an equilateral triangle with sides of one unit in length. We also have 5 points. We can place each point either on the sides of the triangle or within its borders. Show that no matter where these five points are placed within this area, at least two of them will be half a unit or less apart.

Puzzle 4.

The game "Towers of Hanoi" consists of a board with three pegs sticking up from it, with a pyramid of circular disks starting on the left peg. It looks like this:



As a turn, the player is allowed to move a disk from one peg to another. The restriction is that the disk they move must be the top disk on its peg, and that no disk can be placed on a smaller disk. The goal of the game is to move all the disks from the left peg to the right peg.

There is a perfect strategy to this game, which wins it in the minimum number of turns. This minimum number of turns is described by the equation $2^n - 1$, where n is the number of disks. Now imagine the same game, but with a twist. Now you can only move disks to the center peg, or from the center peg, so you cannot go between the right and left pegs.

The question is, with perfect strategy, what is the minimum number of turns to win this new version of the game, with n disks? Also, prove it.

Puzzle 5.

A few years ago I ran across a logic puzzle that really got me exploring and wondering how to solve it. The logic puzzle is called "Blue eyes." In this puzzle there are 200 people on an island.

Half of them have blue eyes, and half have red eyes, but they don't know that. They have been told the following; there are 200 people on the island, at least one of them has blue eyes, if they figure out their own eye color they will leave the island that night at midnight (the how is not important), and that there is no communication available on this island including but not limited to: reflections, speech, writing, pictures, and sign language.

The goal of this puzzle is to figure out how many people will leave the island each night. The night after they learn that at least one person has blue eyes is the first night.

Puzzle 6.

You're walking along in the woods when you come to a split in the path. There is a sign in the fork saying "one path leads to a free luxury resort, and the other leads to a volcano". You want to the luxury resort, but don't know which path is which. Once you choose, you cannot go back. There are also 2 people in front of the fork. One of them always lies, and the other one

always tells the truth. You don't know which one is which. Each of them knows which path leads to the luxury resort. You can ask 1 person 1 question in order to figure out which path to take. What question should you ask?

Puzzle 7.

You have a simple scale -- same as the one in the other problem. It has two sides and it will tell you whether the two sides are equal, or which side is heavier and lighter.

You also have 12 coins. 11 of them are all the same weight and same size, but one coin is fake. It looks and feels the same as the others but it is either slightly heavier or slightly lighter.

If you can only use the scale 3 times, how do you find the fake coin?

Puzzle 8.

A boy needs to go to various stores to buy food for his family. The boy's house and the

stores are positioned on a grid. The boy has a limited amount of time to buy food, after which he must return to his house. At each store, the boy must spend a certain amount of time waiting for food, after which point he will receive a certain amount of food, and he will not be able to return to the store. These quantities are different for each store. The goal is to write a program that will determine the route that gives the most food given a the locations of the stores, the boy's house, and the length of the lines and amount of food at each store.

Puzzle 9.

There are two robots in a room that has only two exits. One of the exits will lead you to freedom while the other will lead to immediate death (you want to be alive). Both the robots know which door is which and will answer your question. You know that one of them always tells the truth and the other always tells lies but you don't know which is which. You're given a single chance to ask only one of them a question and escape the room. What is the question you ask them?

Puzzle 10.

A problem I have run across a few times that I would consider my second favorite, behind the riddle of the man and his fox, goat and cabbage which got me into riddles in the first place, goes something like this:

Across the world, 100 of the wisest man in the world are abducted by aliens without warning. They are then told that the human race is being tested to see if it has grown wise enough to be told the secrets of faster than light travel, and that the outcome of the test decides the fate of the human race. The wise men are then told that in 10 minutes, they will all be lined up with hats that are either black or white on their heads and blinders on so all they can see is the hats of all the men in front of them. Each man, starting with the man in the back, will then have to state the color of his own hat, and if he gets it wrong he will be vaporized. The men are also informed that should any of their number cheat in any way, the entire 100 shall be vaporized, along with the earth. What plan can the men come up with to ensure that the most men possible survive?

Hint: The maximum number that can be saved every time is 99.

Puzzle 11.

Classic paradoxes always spring up around Time Machines. Just a suggestion. In case that one is giving you nightmares (don't worry, most people don't like those ones) I will give you Zeno's Paradox.

Let's say a turtle and a runner are 100m apart, with the runner going past the turtle, and the turtle is running away from the runner. In 10 seconds, the runner goes 50m, and the turtle goes $1\text{m}/20$ seconds. So, after 20 seconds, the turtle and the runner are 1m apart. However, by the time the runner moves 2m ahead, the turtle moved up a little bit. By the time the runner goes that little bit, the turtle moved up a little more! By this logic, the runner never gets ahead from the turtle.

How can you prove this is incorrect? What circumstances can makes this logic correct? If this logic is correct (but not the conclusion made about the runner never passing the turtle) what can you conclude about time? (Infinite points in time?)

Puzzle 12.

A logic puzzle that really got me thinking (and is one of my favorites) is the 5 pirates problem. The scene is as follows. A group of 5 pirates find 100 gold coins and need to divide them up. However, they can't come to an agreement. Then, one of the pirates proposes a scheme. The oldest among them will come up with a scheme to divide up the coins and then everyone will vote on it. If 50% or more people vote yes, then the coins are divided that way. If not, the person who proposed the plan will be thrown overboard and the process will repeat with the second oldest Pirate proposing a plan. Given that pirates are a bloodthirsty bunch and will vote against a plan if they see another way for them to make the same amount of money, what will happen? What would happen if the person who made the plan couldn't vote?

Puzzle 13.

There are 1000 bottles of wine. They are all identical in every aspect except for one bottle, which is poisoned with a lethal toxin. This toxin

cannot be detected in any way, and a single drop of this poisoned wine would kill a human in 10 to 20 hours (no correlation to the dose). What is the least number of slaves needed to identify the poisoned bottle in less than 24 hours?

Puzzle 14.

There are 20 coins on a table, with 10 currently tails and 10 currently heads. You are blindfolded and you are able to feel where the coins are but not if they are heads or tails. by moving and flipping the coins, you must create two sets of 10 coins so that there are the same number of heads and tails in each set. What do you have to do to make this happen?

