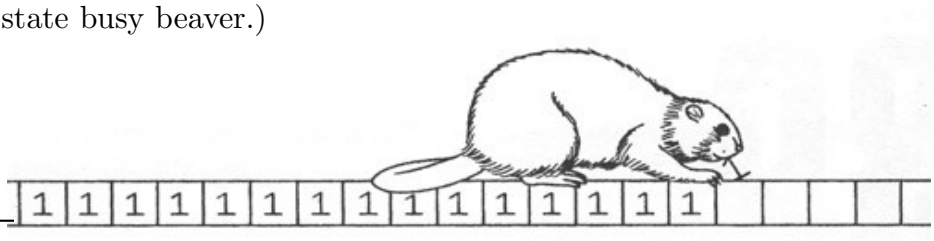
# Busy Beaver Numbers

Define the **n$^{th}$ Busy Beaver number** as follows:

> *BB(n) = The maximum number of steps that an **n-state** Turing machine which **does not loop** can run before it finally halts.*

(I'll show you an example of a 5-state busy beaver.)




## Values of *BB(n)*

| *n* | *BB(n)* |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

## Uncomputability of *BB(n)*

What if we had a Turing machine *BB*, which reads an input *n* and generates the value of *BB(n)*?

Then, we could use this Turing machine to solve the Halting Problem! (How?)*

This proves that ***BB(n)* is uncomputable***:* we cannot have a Turing machine which solves it, because we know we can't solve the Halting Problem.

**Hints:*

1. *We can run a finite number of Turing machines in parallel, step-by-step.*
2. *There are a finite number of Turing machines with n states.*
3. *We can count how many of the Turing machines halt, and compare this to BB(n).*