Problems

Determine whether the following sets are *countably infinite* or *uncountably infinite*. Try to solve as many as you can! Discuss ideas with your neighbors.

1. **The # of finite binary strings** — Countable ∞ / Uncountable ∞

2. **The # of lists of finite binary strings** — Countable ∞ / Uncountable ∞

3. **The # of positive integers** (1, 2, 3, 4, ...) — Countable ∞ / Uncountable ∞

4. **The # of pairs of positive integers**
   ((1,1), (1,2), ... (2,1), (2,2), ... (3,1), ...) — Countable ∞ / Uncountable ∞

5. **The # of lists of positive integers** — Countable ∞ / Uncountable ∞

6. **The # of integers** (... -2, -1, 0, 1, 2, ...) — Countable ∞ / Uncountable ∞

7. **The # of real numbers between 0 and 1**
   (7.1248..., -2.99212..., 1.05127..., etc.)
   (This means all numbers, even ones
   with an infinite decimal, such as π/4.) — Countable ∞ / Uncountable ∞

8. **The # of rational numbers between 0 and 1**
   (Anything that can be written as
   "*an integer ÷ an integer*".)
   (1.3333, 77.25, 5.0, -37.1, etc.) — Countable ∞ / Uncountable ∞

9. **The # of real numbers** — Countable ∞ / Uncountable ∞

10. **The # of rational numbers** — Countable ∞ / Uncountable ∞

<u>More Challenging Problems</u>

11. **The # of pairs of real numbers**

12. **The # of integers between 0 and 1**

13. **The # of infinite binary strings**

14. **The # of lists of lists of integers**

15. **The number of lists of lists of lists of integers**

A formative day for Georg Cantor.