Turing Machines: Problemsolving

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(from Sipser, chapter 3)

1 PROBLEM 1

Recall that a language L is *decidable* if there exists a Turing machine M which *decides* it: $M(x)$ = ACCEPT for every $x \in L$, $M(x)$ = REJECT for every $x \notin L$.

Show that the collection D of decidable languages is *closed* under each of the below operations. ("Closed" means that if you apply the operation to any language or languages in D, the result is still in D.)

1.1 UNION

1.2 CONCATENATION

1.3 STAR

1.4 COMPLEMENTATION

1.5 INTERSECTION

2 PROBLEM 2

Recall that a language L is *recognizable* if there exists a Turing machine M which *recognizes* it: $M(x)$ = ACCEPT for every $x \in L$, but $M(x)$ = either REJECT or LOOP for every $x \notin L$. If x is not in L, then M is not guaranteed to halt at all.

Show that the collection R of recognizable languages is closed under each of the below operations.

2.1 UNION

2.2 CONCATENATION

2.3 STAR

2.4 INTERSECTION

3 PROBLEM 3

Show that a language L is decidable if-and-only-if there is a Turing machine E, an “enumerator”, which enumerates the language in alphabetical order.

Remember, since this is if-and-only-if, you must prove both ways:

3.1 IF THERE IS A MACHINE D WHICH DECIDES L, THEN THERE IS AN ENUMERATOR E WHICH ENUMERATES L.

3.2 IF THERE IS AN ENUMERATOR E WHICH ENUMERATES L, THEN THERE IS A MACHINE D WHICH DECIDES L.