Pumping Lemma Examples

Chelsea Voss (csvoss@mit.edu)

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(These examples are not my own; they are from 6.045 Pset 1, Spring 2013, by Scott Aaronson.)

1 PROBLEM 1

Let $L \subseteq \{a, b\}*$ be the language consisting of all *palindromes*: that is, strings like *abba* that are the same backwards and forwards. Show that L is not regular. (You can use either the pigeonhole principle or the Pumping Lemma from Sipser's book.)

1.1 ANSWER

For contradiction, assume L is regular. Then, by the Pumping Lemma, there must exist some pumping length p, such that for any $w \in L$ where $|w| \geq p$, w satisfies the three conditions of the Pumping Lemma.

Let $w = 0^{2p}1^{2p}1^{2p}0^{2p} \in L$. By the conditions of the Pumping Lemma, there must be some substring of w that we can repeat as many times as we want such that the resulting string is still in L; furthermore, this substring must occur within the first p characters of w, which means that it occurs entirely within the first block of 0's in w. But then the resulting string will have more 0's in the beginning than in the end and thus not be a palindrome anymore. This is a contradiction, so the original assumption that L was regular must be false!

2 PROBLEM 2

Let $L = \{1^n | n \text{ is prime}\}$. Show that L is not regular. (You can use the fact that there are infinitely many prime numbers.)

2.1 ANSWER

For contradiction, assume L is regular. Then, by the Pumping Lemma, there must exist some pumping length p, such that for any $w \in L$ where $|w| \geq p$, w satisfies the three conditions of the Pumping Lemma.

Since there are infinitely many primes, we know that there must be some prime $k > p$; let $w = 1^k$.

Let the part of w that we can pump up be of some length n, such that when we pump up a times, we get a new string 1^{k+an}. To avoid a contradiction, it must be the case that $\forall a$ (that is, for all a), $k + an$ is prime. However, if

$a = k$, then $k + an = k + kn = (n+1)k$, which is divisible by k, and so can't be prime. Thus we have a contradiction and so L is not regular!