Context-Free Grammars: Answer Key

Chelsea Voss (csvoss@mit.edu)

July 13, 2014

(from 6.045 Pset 2, Spring 2013)

There are many correct answers – the ones given below are examples.

1 PROBLEM 1

Show that the following languages are context-free, by giving context-free grammars for them.

1.1 $L = \{$'', 'abb', '$aabbbb$', '$aaabbbbbb$', ...$\}$. THAT IS, $L = \{a^n b^{2n}$, WHERE $n \geq 0\}$.

Terminals: $[a, b]$
Variables: $[A]$
Start: A
Rules: $A \to aAbb$
$A \to \varepsilon$

1.2 THE LANGUAGE L THAT CONSISTS OF ALL STRINGS OF *balanced parentheses*: FOR EXAMPLE, (()())()(()) IS IN L, BUT ())(() IS NOT IN L.

Terminals: $[(,)]$
Variables: $[A]$
Start: A
Rules: $A \to A(A)A$
$A \to \varepsilon$

1.3 $L = \{x \in \{a, b\}^*$, WHERE x CONTAINS AN EQUAL NUMBER OF a'S AND b'S $\}$

Terminals: $[a, b]$
Variables: $[A]$
Start: A
Rules: $A \to AaAbA$
$A \to AbAaA$
$A \to \varepsilon$

1.4 $L = \{x \in \{a,b\}^*$, WHERE x CONTAINS MORE a'S THAN b'S $\}$

Terminals: $[a, b]$
Variables: $[A]$
Start: A
Rules: $A \to AaAbA$
$A \to AbAaA$
$A \to AaA$
$A \to \varepsilon$

2 PROBLEM 2

Show that context-free languages are *"closed under union"*: that is, if A and B are CFLs, then $A \cup B$ is a CFL also.

Proof. Suppose that we have one grammar describing CFL A, where the grammar starts with some variable named V_A, and one grammar describing CFL B with corresponding start variable V_B.

Assume that the grammars for A and B use different, unique names for all of their variables, so that they don't interfere with each other. Let V_0 be a completely new variable that is not used in either context-free grammar.

Then, we can construct a context-free grammar describing the language $A \cup B$ as follows.

Terminals: (terminals of CFLs A and B
Variables: (variables of CFGs A and B)
Start: V_0
Rules: $V_0 \to V_A$
$V_0 \to V_B$
(rules of CFG A)
(rules of CFG B)