

Finite Automata and Regular Languages – Problems

Problem 1. Regular Operations

The class of regular languages is closed under the concatenation operation. This means that if A and B are both regular languages, then $A \circ B$ is also a regular language.

Prove this by drawing a *rough, approximate* diagram for a finite automaton that recognizes the language $A \circ B$, using the finite automata for A and B as building blocks.

The class of regular languages is closed under the union operation. This means that if A and B are both regular languages, then $A \cup B$ is also a regular language.

Prove this by drawing a *rough, approximate* diagram for a finite automaton that recognizes the language $A \cup B$, using the finite automata for A and B as building blocks.

*Hint: This will be substantially easier if you use a **nondeterministic** finite automaton.*

The class of regular languages is closed under the star operation. This means that if A is a regular language, then A^* is also.

Prove this by drawing a *rough, approximate* schematic for a finite automaton that recognizes the language A^* , using the finite automaton for A as a building block.

Problem 2. Regular Expressions

Regular languages can be written down using the regular operations, as an alternative to drawing out the entire state diagram. This is called a *regular expression*.

For example, this is the regular expression for the regular language consisting of all strings that only consist of 0s:

$$0^*$$

This is the regular expression for the regular language consisting of 01, 0101, 010101, and so on:

$$0 \circ 1 \circ (0 \circ 1)^* \quad (\text{or, abbreviated: } 01(01)^*)$$

Parentheses are allowed, in order to group items together.

- a) Write the regular expression for the language consisting of any number of 0s, followed by any number of 1s.

- b) Write the regular expression for the language consisting of all strings that begin and end with a 0.

- c) Write the regular expression for the language consisting of all strings which have an even number of 1s.

- d) Write the regular expression for the language consisting of only the string 100101 or the string 0111000, but no other strings.

- e) Write the regular expression for the language consisting of all strings which have an even number of 1s or an even number of 0s (or both).