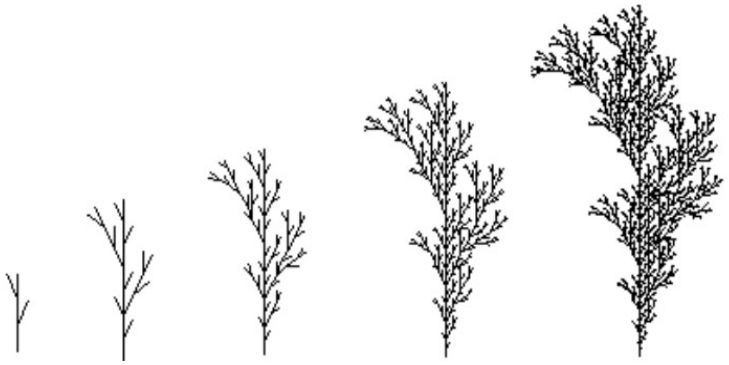
Fractals and L-systems

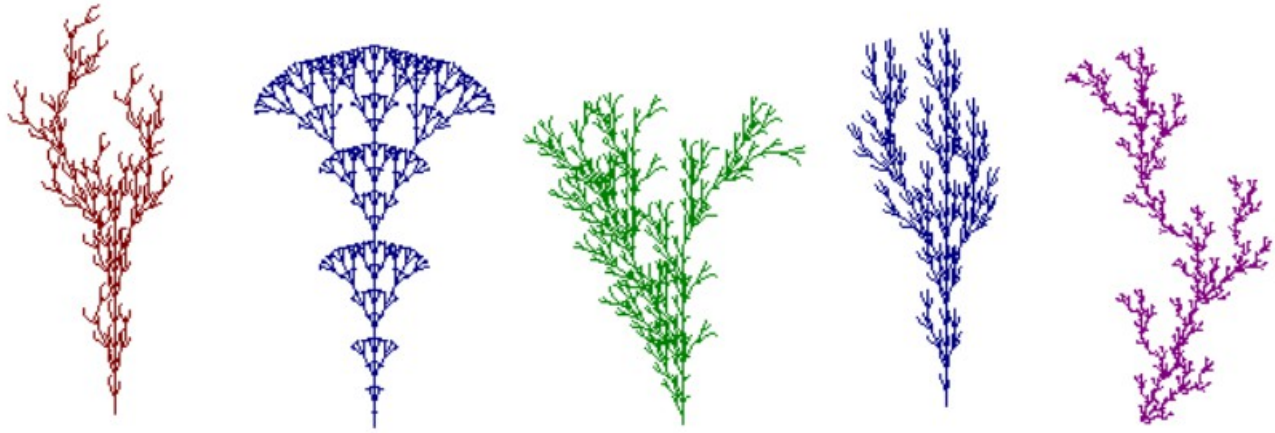
"L-systems are a mathematical formalism proposed by the biologist Aristid Lindenmayer in 1968 as a foundation for an axiomatic theory of biological development. Two principal areas include generation of fractals and realistic modelling of plants.



Central to L-systems, is the notion of ***rewriting****, where the basic idea is to define complex objects by successively replacing parts of a simple object using a set of rewriting rules or productions. The rewriting can be carried out* ***recursively.***

The most extensively studied and the best understood rewriting systems operate on character strings. Chomsky's work on formal grammars (1957) spawned a wide interest in rewriting systems. Subsequently, a period of fascination with syntax, grammars and their application in computer science began, giving birth to the field of formal languages.

Aristid Lindenmayer's work introduced a new type of string rewriting mechanism, subsequently termed L-systems. The essential difference between Chomsky grammars and L-systems lies in the method of applying productions. In Chomsky grammars productions are applied sequentially, whereas in L-systems they are applied in ***parallel****, replacing simultaneously all letters in a given word. This difference reflects the biological motivation of L-systems. Productions are intended to capture cell divisions in multicellular organisms, where many division may occur at the same time."*



Simple Cells

Think of this as a system for modeling the growth of a 1-dimensional line of algae. Cell type A can divide into an A next to a B, while cell type B must mature into an A before it can divide.

variables: A, B
start: B
rules: (A → AB), (B → A)

Fill in the blanks for each generation.

t = 0: B

t = 1: A

t = 2: _____

t = 3: _____

t = 4: ABAAB

t = 5: _____

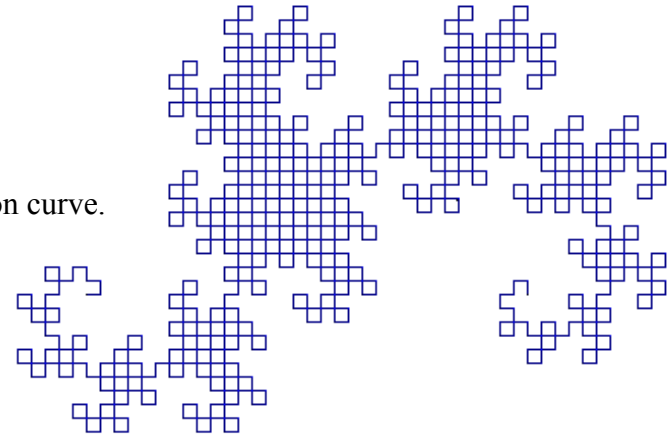
t = 6: _____

t = 7: ABAABABAABAABABAABABA

t = 8: ABAABABAABAABABAABABAABAABABAABAAB

Count the number of cells in each generation... do you notice anything? (Optional: Can you try to prove why this is happening?)

Dragon Curve



This system is for generating the rules to draw a dragon curve.

variables: A, B
start: B
rules: (B → B–A), (A → A+B)
after-rules: A or B means "draw forward"
+ means "turn left 90°"
– means "turn right 90°"

Fill in the blanks for each generation.

t = 0: _____

t = 1: _____

t = 2: _____

t = 3: _____

t = 4: _____

t = 5: _____

Try to draw a Dragon Curve using one of the results that you get!